Exercise 4

Find the linearization $L(x)$ of the function at a.

$$f(x) = 2^x, \quad a = 0$$

Solution

Start by finding the corresponding y-value to $x = 0$.

$$f(0) = 2^0 = 1$$

Then find the slope of the tangent line to the function at $x = 0$ by computing $f'(x)$,

$$\begin{aligned} f'(x) &= \frac{d}{dx}(2^x) \\ &= \frac{d}{dx}(e^{\ln 2^x}) \\ &= \frac{d}{dx}(e^{x \ln 2}) \\ &= e^{x \ln 2} \cdot \frac{d}{dx}(x \ln 2) \\ &= e^{x \ln 2} \cdot (\ln 2), \end{aligned}$$

and plugging in $x = 0$.

$$f'(0) = e^0 \cdot (\ln 2) = \ln 2$$

Now use the point-slope formula to obtain the equation of the line going through $(0, 1)$ with slope $\ln 2$.

$$\begin{aligned} y - f(0) &= f'(0)(x - 0) \\ y - 1 &= (\ln 2)x \\ y &= (\ln 2)x + 1 \end{aligned}$$

Therefore, the linearization of the function $f(x)$ at $a = 0$ is

$$L(x) = (\ln 2)x + 1.$$

Below is a plot of the function and the linearization at $a = 0$ versus x.